

TESTS OF BROAD-CRESTED WEIRS

BY

JAMES G. WOODBURN

**A dissertation submitted in partial fulfillment of the requirements
for the Degree of Doctor of Philosophy in the
University of Michigan**



AMERICAN SOCIETY OF CIVIL ENGINEERS

INSTITUTED 1852

PAPERS AND DISCUSSIONS

This Society is not responsible for any statement made or opinion expressed in its publications.

TESTS OF BROAD-CRESTED WEIRS

BY JAMES G. WOODBURN,* ASSOC. M. AM. SOC. C. E.

(Reprinted from *Proceedings*, September, 1930)

SYNOPSIS

This paper gives the results of 305 tests made during 1928-29 at the University of Michigan, Ann Arbor, Mich., on broad-crested weirs of various designs in a rectangular wooden flume 2 ft. wide. The crests varied in breadth from 10 to 15.5 ft., and in slopes and combinations of slopes from level to 0.085. The range of head was from 0.5 ft. to 1.5 ft. and of volume of flow from 2.0 to 11.0 cu. ft. per sec.

The purpose of the tests was to obtain experimental data in regard to certain advantages suggested in favor of the broad-crested weir over the sharp-crested weir as a device for measuring flowing water. One advantage established by the tests is that the tail-water can be backed up to an elevation only slightly lower than the water surface at the down-stream end of the weir without changing appreciably the relation between head-water elevation and discharge. Moreover, if the slope of the weir is such that a hydraulic jump can be made to occur on the crest, nearly all the initial head can be recovered, still without changing the head-water-discharge relation.

This ability to operate when submerged adapts the broad-crested weir for use under conditions where head is at a premium, as, for example, in the measurement of irrigation water. The broad-crested weir, moreover, even when operating under a considerable degree of submergence, requires the measurement of head at only one point and is thus preferable to the submerged sharp-crested weir and the Venturi flume, which require the measurements at two points.

Another advantage is the possibility of causing flow to occur at critical depth at some predetermined point on the crest at which the depth can be readily measured. The discharge is then a simple function of the depth, independent of the confusing effects of velocity of approach.

Previous discussions of this subject had indicated that critical depth might be expected to occur at the change in grade between slopes of channel one of

NOTE.—Written discussion on this paper will be closed in **January, 1931**, *Proceedings*.

* Associate Prof. of Hydr. Eng., State Coll. of Washington, Pullman, Wash.

which would maintain flow above, and the other below, the critical depth. To investigate this theory tests were made of weirs formed of 10 ft. of level crest followed by $3\frac{1}{2}$ ft. of apron with a small down-stream slope, the entrance to the weir being rounded.

These tests showed that the location of critical depth is affected by waves on the crest. The conformation of these waves is, in turn, affected by the volume of flow. With flows up to about 8 cu. ft. per sec., critical depth in a number of tests occurred at the change in grade, provided the slope of the apron was not too steep; but in other tests the waves caused the position of critical depth to move short distances up or down stream. When the flow was increased to more than 8 cu. ft. per sec. the position of critical depth moved from 1 to 3 ft. up stream from the change in grade.

Waves occurred at certain volumes of flow with every weir model tested in which part or all of the crest was level. If the crest was given a slope sufficient to maintain flow without waves, critical depth was found to occur, as a rule, only at some point of the initial drop at the entrance to the weir, at which the water surface sloped steeply and a small error in selection of the point of measurement would cause considerable error in the measured depth.

The tests showed that there was no single location of critical depth for all volumes of flow. Some interesting relations, however, were brought out between the design of the weir, the number of waves on the crest, and the location of critical depth.

Much information was obtained in regard to weir coefficients and to the factors affecting their variation. Coefficients as high as 3.08 were reached with a weir with rounded entrance followed by a down-stream slope of 0.026. This slope of crest permitted the formation of the hydraulic jump and the recovery of nearly all the initial head.

INTRODUCTION

Few previous tests have been made of broad-crested weirs of the designs described in this paper. Experiments have been made by Bazin, Blackwell, Fteley and Stearns, the U. S. Geological Survey, and others, on many designs of dams and weirs not sharp-crested, but only a small part of their investigations has dealt with weirs with broad level crests or with broad crests having a small down-stream slope.

However, recent discussions and tests on the subject of the critical depth and hydraulic jump* have suggested certain advantages which the broad-crested

* "The Hydraulic Jump in Open-Channel Flow at High Velocity," by K. R. Kennison, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 338; "Theory of Hydraulic Jump and Back-Water Curves," by S. M. Woodward, M. Am. Soc. C. E.; also "The Hydraulic Jump as a Means of Dissipating Energy," by R. M. Riegel and J. C. Beebe, The Miami Conservancy Dist., Tech. Repts., Pt. III, 1917; "The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," by Julian Hinds, M. Am. Soc. C. E., *Engineering News-Record*, November 25, 1920; "The Improved Venturi Flume," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), particularly discussions by Julian Hinds and J. C. Stevens, Members, Am. Soc. C. E., p. 841 *et seq.*; "Berechnung der Wasserspiegellage," by P. Boess, *Karlsruhe Forschungsheft No. 284*, Verein Deutscher Ingenieure, Berlin, 1927; "The Hydraulic Design of Flume and Siphon Transitions," by Julian Hinds, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1423.

weir may have over the sharp-crested weir as a device for measuring flowing water. These suggested advantages are:

1.—Its adaptability to the use of the hydraulic jump, to reduce to a minimum the head lost in the weir; and

2.—The possibility of producing flow at critical depth with a resulting simple relation between depth and discharge which is independent of the confusing effects of velocity of approach.

PURPOSE AND SCOPE OF THE TESTS

The purpose of the tests was to obtain experimental data in regard to these two suggested advantages of the broad-crested weir. The breadth of crest of the weirs tested varied from 10 to 15.5 ft., with various down-stream slopes and combinations of slopes from level to 0.085. The range of heads was from 0.5 ft. to 1.5 ft. and of volume of flow from 2 to 11 cu. ft. per sec. These and other details of the tests will be given in the summaries of the experiments.

Coefficients of discharge of various weir models were determined both with free over-fall and with the weir submerged. The data thus obtained permitted a study of the effect of submergence, of the effect on the coefficient of various factors in the design of the weir, and of the variation of the coefficient with the head. Profiles of the water surface were made with different weir models to permit a study of the factors affecting the position of critical depth, with a view to suggesting if possible a design of weir which would produce flow at critical depth at the same point on the crest for all volumes of flow.

NOTATION

The following notation is used in the paper:

H = head on the weir, in feet.

H_a = total head, in feet, at any point in a flowing stream.

h_v = head due to velocity of approach, in feet.

D = depth, in feet.

D_a = depth in channel of approach, in feet.

D_c = critical depth, in feet.

V = velocity, in feet.

v_a = velocity of approach, in feet.

Q = discharge, in cubic feet per second.

Q_1 = discharge, in cubic feet per second, per foot of width.

C = weir coefficient.

g = acceleration due to gravity, = 32.16 cu. ft. per sec. per sec.

α = velocity-head coefficient, correcting for unequal distribution of velocities in cross-section; here considered = 1.0.

REVIEW OF CRITICAL DEPTH THEORY

The total head in any point of a flowing stream is:

$$H_a = D + \alpha \frac{V^2}{2g}$$

Considering $\alpha = 1.0$.

$$V = \sqrt{2g(H_a - D)}$$

Then,

$$Q_1 = D \sqrt{2g(H_a - D)}$$

Critical depth, by definition, is the depth at which, for any given total head, the discharge is a maximum. Differentiating Q_1 with respect to D and equating to zero,

$$H_a = \frac{3}{2} D$$

Substituting,

$$Q_1 = \sqrt{g} (D_c)^{\frac{3}{2}}$$

The discharge is, therefore, a simple function of the critical depth; the velocity of approach is not involved. Thus, if the critical depth can be made to occur at any known point where it can be measured accurately, a simple method of determining the discharge is available.

The effect of error in measurement of D_c may be shown by differentiating Q_1 with respect to D_c and dividing; that is,

$$d Q_1 = \frac{3}{2} \sqrt{g} (D_c)^{\frac{1}{2}} d D_c$$

$$\frac{d Q_1}{Q_1} = \frac{3}{2} \frac{d D_c}{D_c}$$

Thus, for any percentage, p , of error in D_c , the computed Q is in error $1.5 p$ per cent. This degree of accuracy is the same as that of the usual equations for rectangular weirs (aside from velocity of approach) and is greater than that of the equations for V-notch weirs, where an error of $p\%$ in measuring the head results in an error of $2.5p\%$ in the computed Q .

THE LABORATORY

Details of the hydraulic laboratory are shown in Figs. 1 and 3. The quantity of water flowing in the system was controlled by the gate valve at the upper end of the 15-in. pipe line.

The 90° V-notch weir by which the discharge was measured, was calibrated by means of the weighing tanks. The methods and results of this calibration are described under "Measurement of Discharge".

Weir.—At a point 32 ft. from the head end of the flume, the bottom was stepped up 1.75 ft. (Fig. 1), and carried for 10 ft. at this elevation. This formed a weir crest 10 ft. broad, with square-cornered entrance. Series *A* includes the tests of this weir. Rounded entrances of various curvatures were obtained by means of laminated wooden blocks set on bases of proper height and thickness against the up-stream end of the weir.

Variation in slope was obtained by a false bottom 12 ft. long. This bottom was given the desired slope by laying it on shims on the 10-ft. level section. Series *DD* and *DE* give the results obtained with two different slopes of crest.

Combinations of level and sloping crests were obtained by means of a platform 3½ ft. long, used with either the permanent level section or the sloping false bottom. With the former the platform was used as a down-stream apron with three different slopes (Series *DA*, *DB*, and *DC*) while with the sloping bottom the platform was placed ahead to form a level section of crest (Series *DF* and *DG*).

The flume and weir were built in August, 1928. Just prior to the tests which began in November, the crest of the permanent weir was planed to a true level surface, and the sides of the flume, as well as the weir crest, were scraped and sand-papered to give as smooth a surface as possible. The surfaces of the false bottom and short platform were also made smooth.

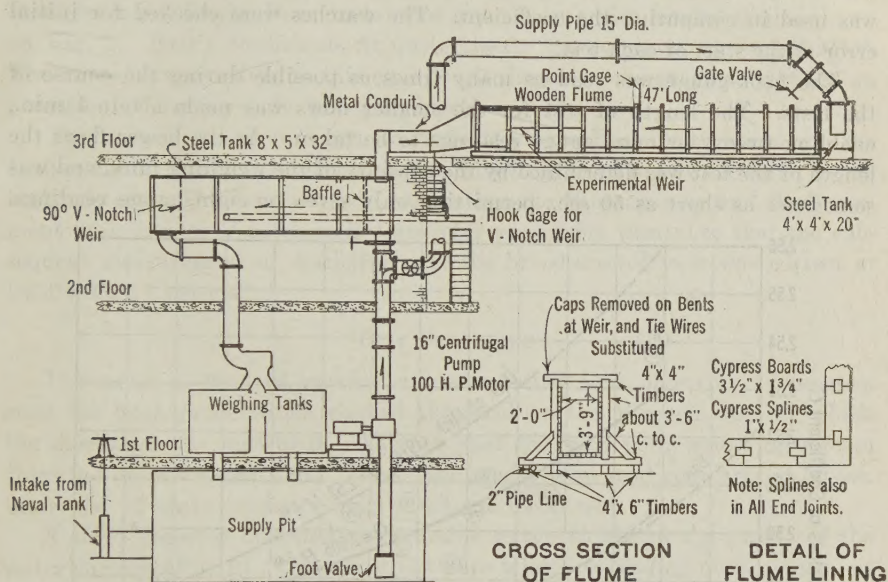


FIG. 1.—DETAILS OF EXPERIMENTAL FLUME.

Aeration.—Five 1-in. holes to provide aeration of the nappe were bored in the wooden bulkhead forming the down-stream end of the permanent weir section. These holes were in a horizontal line 5 in. below the weir crest. Wooden plugs were used to stop the holes when required. As a general rule, the plugs were removed and the nappe was aerated in tests in which the back-water from the fall did not rise high enough to cause leakage through the holes. The nappe was thus aerated for flows up to about 7 cu. ft. per sec.

Measuring Elevation of Water Surface.—The elevation of the head water was measured by a point-gauge mounted in the center of the flume on the cross-cap of one of the bents (Figs. 1 and 3). The distance from the up-stream edge of the weir to the gauge was 7 ft. When the weir was extended up stream by the addition of the 3½-ft. platform, the gauge was moved up to the next bent. Profiles of the water surface over the weir were taken with another point-gauge mounted on a light channel iron. This movable gauge was supported by parallel channels attached to the tops of the posts.

Measurement of Discharge.—The discharge over the broad-crested weir was measured by means of a 90° V-notch weir which was calibrated gravimetrically just previous to the tests. The methods and results of this calibration will be discussed here briefly.

The flying-start-and-stop method was used in measuring the water. With the flow steady and the water discharging through the weighing tank, the

weight on the scale-beam was set to a small initial reading and the outlet valves of the weighing tank were closed. At the instant the beam rose to the cross-bar the two watches were started. The weight was then set ahead to the desired final quantity, and the watches were stopped when the beam rose to the bar again. The average of the watch readings, corrected for initial error, was used in computing the coefficient. The watches were checked for initial error at the start of each test.

The hook-gauge was read as many times as possible during the course of the tests. The length of test for the smaller flows was made about 4 min., enabling twenty or more gauge readings to be taken. At the larger flows the length of the test was determined by the capacity of the weighing tank, and was sometimes as short as 50 sec., permitting only seven or eight gauge readings.

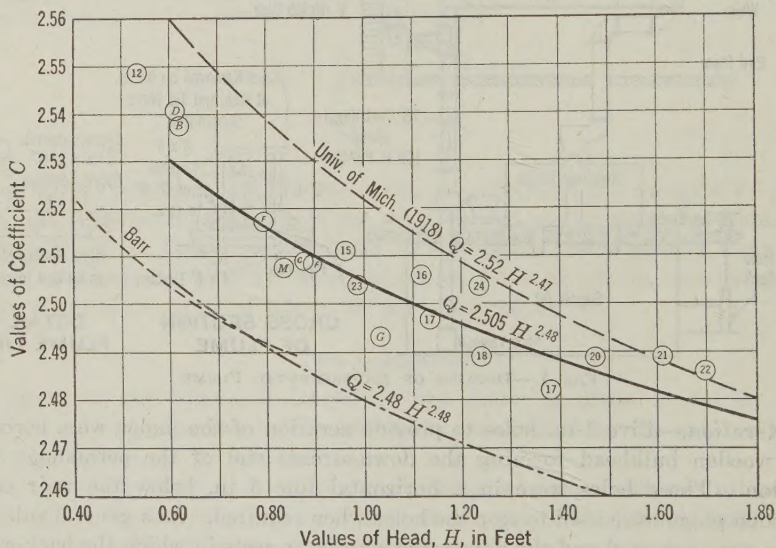


FIG. 2.—CALIBRATION TESTS OF 90° V-NOTCH WEIR: VARIATION OF C WITH H IN $Q = CH^{2.5}$ FOR THE EQUATIONS SHOWN.

The volume of flow, in cubic feet per second, was obtained by using 62.40 lb. as the weight of 1 cu. ft. of water. The computed volume of flow and the observed head on the weir corrected for zero reading of the gauge, were used to compute the coefficient, C , in the equation, $Q = CH^{\frac{5}{2}}$. Values thus obtained for C are plotted against values of H in Fig. 2, which shows the usual decrease in coefficient with increase in head. The equation, $Q = CH^{\frac{5}{2}}$, is awkward to use because it involves two variables. By making a slight change in the exponent of H , a suitable equation can be derived in which C is constant. The equation best fitting the points on the graph was adopted for use, namely, $Q = 2.505 H^{2.48}$. The curve representing this equation (Fig. 2) was obtained by plotting values of C against H . C is computed from the equation, $C = \frac{Q}{H^{\frac{5}{2}}}$.

Q having been computed from the equation, $Q = 2.505 H^{2.48}$. Thus, the curve and the test results are comparable graphically.

Curves of two other series of experiments with 90° V-notch weirs are also shown on Fig. 2. The equation, $Q = 2.52 H^{2.47}$, was obtained as the result of experiments performed at the University of Michigan in 1918. In Scotland, some years previously, Barr made some experiments with results as also shown on Fig. 2. Barr's coefficients fit quite closely the equation, $Q = 2.48 H^{2.48}$. The curve corresponding to this equation is drawn on the graph to serve as an extension of Barr's results to higher heads.

The writer's equation is seen to lie between the Barr experiments and the University of Michigan equation, but approaching more nearly the latter at the higher heads. The care taken in calibrating the V-notch weir and the agreement with former V-notch weir tests offer reasonable assurance that the subsequent measurement of discharge over the broad-crested weir was correct at least within 1 per cent.

TEST PROCEDURE

The usual method of making a test required four observers. One man read the hook-gauge which showed the head on the V-notch weir by which the discharge was measured. Another read the fixed point-gauge up stream from the broad-crested weir. These two gauges were read regularly at 30-sec. intervals. Twenty readings constituted the usual test.

A third observer operated the movable gauge to obtain the profile of the water surface (Fig. 3). Cross-sections were taken, proceeding down stream, at as many points as were required to permit drawing an accurate profile. The usual cross-section consisted of three readings, one at the center and one 3 in. from each side. The average of these readings was used in plotting the profile.

The fourth man kept notes for the second and third observers. The man reading the hook-gauge recorded his own notes.

A number of tests were made merely to determine coefficients of discharge without taking profiles of the water surface. These tests required an observer on the hook-gauge, one on the fixed point-gauge, and a recorder.

METHOD OF COMPUTING WEIR COEFFICIENTS

The weir coefficient for each test was computed by the method outlined by Horton in his treatise on "Weir Experiments, Coefficients, and Formulas."* The steps in the computation will be given briefly.

The average reading of the hook-gauge, corrected for the zero reading of the gauge, was used to find the discharge, Q , from a table prepared by the writer, based on the formula already stated, $Q = 2.505 H^{2.48}$.

The average reading of the fixed point-gauge, corrected for the zero reading, was used as the observed head, H , on the weir.

The depth of water in the channel of approach,

$$D_a = H + \text{the height of the weir}$$

* U. S. Geological Survey, *Water Supply and Irrigation Paper No. 200*, pp. 66-67.

Then,

$$v_a = \frac{Q}{2 D_a}$$

$$h_v = \frac{v_a^2}{2 g}$$

$$H_a = H + h_v$$

$$C = \frac{Q}{2 H_a^{\frac{3}{2}}}$$

$$D_c = \left(\frac{Q_1}{\sqrt{g}} \right)^{\frac{2}{3}} = \left(\frac{Q}{2 \sqrt{g}} \right)^{\frac{2}{3}} = 0.1982 Q^{\frac{2}{3}} \text{ (see page 1586).}$$

D_c was found from a table prepared by the writer for a rectangular flume, 2 ft. wide, giving critical depths for values of Q from 0 to 12 cu. ft. per sec.

OUTLINE OF TESTS

An outline of the 305 tests of broad-crested weirs, with the series designations, in the chronological order of their performance, is as follows:

Tests of weirs with level crest, 10 ft. broad, with varying curvature of entrance:

Square cornered entrance.....	37 tests, Series <i>A</i>
Rounded entrance, 2-in. radius.....	13 tests, Series <i>B</i>
Rounded entrance, 3-in. radius.....	11 tests, Series <i>C</i>
Rounded entrance, 6-in. radius.....	26 tests, Series <i>D</i>
Rounded entrance, 3 by 6-in. ellipse, long axis horizontal.....	10 tests, Series <i>E</i>
Rounded entrance, 8-in. radius.....	12 tests, Series <i>F</i>
Rounded entrance, 4 by 8-in. ellipse, long axis horizontal.....	12 tests, Series <i>G</i>

Tests of weirs from 12 to 15.5 ft. broad, with various slopes and combinations of slopes. Entrance rounded on 6-in. radius:

10-ft. level, followed by 3.5 ft. of 0.015 slope.....	25 tests, Series <i>DA</i>
10-ft. level, followed by 3.5 ft. of 0.004 slope.....	36 tests, Series <i>DB</i>
10-ft. level, followed by 3.5 ft. of 0.085 slope.....	8 tests, Series <i>DC</i>
12 ft. of 0.004 slope.....	33 tests, Series <i>DD</i>
12 ft. of 0.026 slope.....	36 tests, Series <i>DE</i>
3.5-ft. level, followed by 12 ft. of 0.026 slope.....	33 tests, Series <i>DF</i>
3.5-ft. level, followed by 12 ft. of 0.004 slope.....	13 tests, Series <i>DG</i>

Total tests 305

ANALYSIS OF RESULTS

The results will be discussed under three heads:

- 1.—Effect of submergence on the weir coefficient.
- 2.—Location of critical depth.
- 3.—Variation of coefficient with head for different weir models.

Effect of Submergence on Coefficient.—The results of fifty-four pairs of tests which show the effect of submergence on the weir coefficient, are sum-

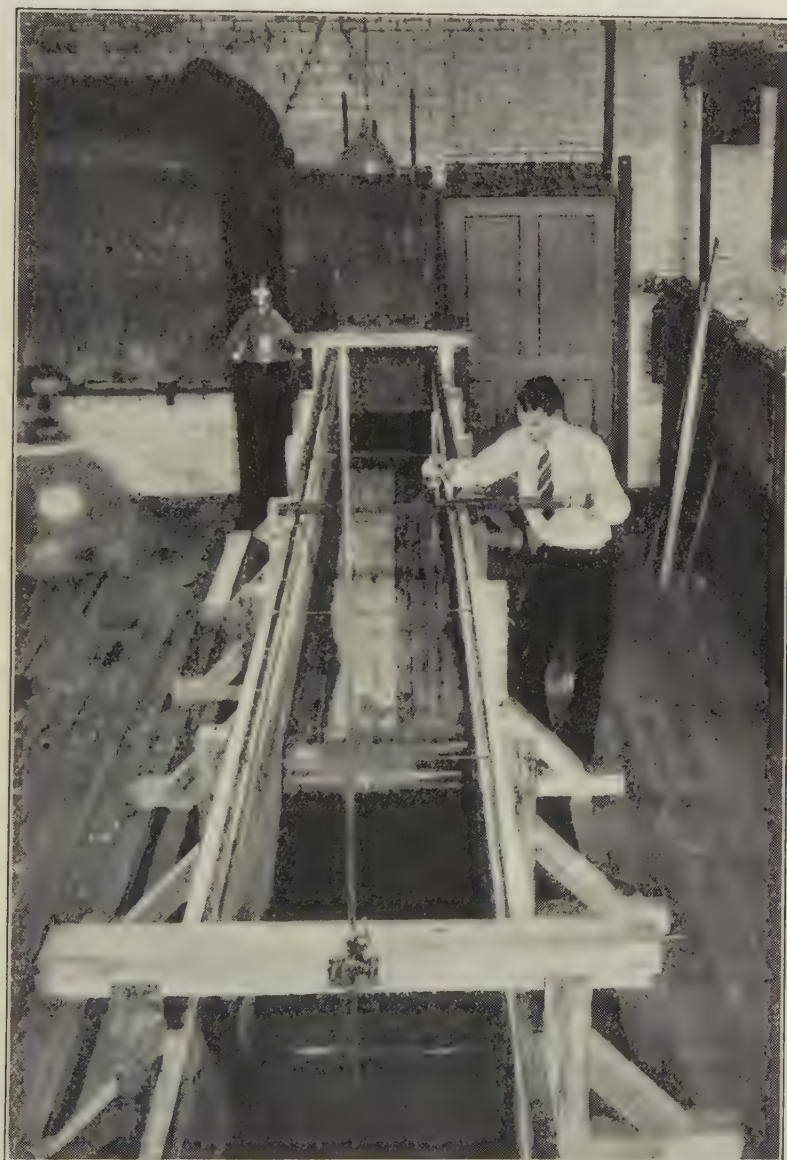


FIG. 3.—SET-UP AND DETAILS OF HYDRAULIC FLUME.



marized in Table 1. These tests were made on eight different weir models and covered the entire range of head from 0.5 ft. to 1.5 ft. Each pair, in nearly every case, was performed consecutively with no change between tests in the quantity of water flowing.

Either with or without the hydraulic jump, the average difference in the coefficients of the weirs submerged and with free over-fall was found to be inappreciable. In thirty-two pairs of tests in Table 1 (Part I), in which the jump could not be produced when the weir was submerged, the coefficient for the submerged weirs was, on the average, 0.219% less than for the weirs with free over-fall; while in twenty-two pairs of tests in Table 1 (Part II), in which the jump was formed on the crest when the weir was submerged, the coefficient for the submerged weirs was, on the average, 0.081% less than for the weirs with free over-fall. In the fifty-four pairs of tests in both parts, the average coefficient for the submerged weirs was 0.163% less than for the weirs with free over-fall.

The depths of submergence are shown in the profiles of water surface in Fig. 4. In the tests in which the slope of the crest was steep enough to permit the formation of the hydraulic jump (Series *DE* and *DF*), the loss of head over the weir was limited to about 0.2 ft. at the smallest flow and about 0.3 ft. at the largest flow. In the tests of weirs with slopes so flat that the jump could not be formed (Series *DA*, *DD*, and *DG*), the tail-water could be raised only to an elevation somewhat below the water surface at the downstream end of the weir without noticeably affecting the conditions of flow over the weir, and the loss of head was from about 0.3 ft. for the smallest flow to about 0.6 ft. for the largest flow.

The attempt was made at the start of each test with the weir submerged to back up the tail-water to an elevation just under the maximum which would leave the weir coefficient unchanged. If at the start of a test, however, the tail-water was very close to the permissible elevation, slight fluctuations in volumes of flow might have caused this elevation to be exceeded momentarily with a resulting change in the conditions of flow over the weir and a decrease in the weir coefficient. The permissible elevation of the tail-water was purposely approached more closely in the tests in which the jump could not be formed than in those in which it could be formed, and as a result was probably exceeded more often in the former case than in the latter. This fact is reflected by the averages of Table 1 (Parts I and II) which show a difference in coefficients of weirs submerged and with free over-fall somewhat larger when the jump could not be formed than when it could be formed.

A typical example of the change in conformation of the water surface over a submerged weir without the jump when the tail-water rises through this permissible elevation is shown by Tests 4 and 5 of Series *DD* (Fig. 4). The volume of flow in the two tests was practically the same; but a small closure of the down-stream gate between tests caused a rise in the tail-water, with a marked change in profile of the water surface over the weir and a rise in the head-water. A change of this nature, although much smaller in extent and caused by a slight fluctuation in the volume of flow instead of by closure of the gate, doubtless occurred during those six tests of Table 1 (Part I) in

TABLE 1.—TESTS OF BROAD-CRESTED WEIRS: COMPARISON OF COEFFICIENTS OF SUBMERGED WEIRS AND WEIRS WITH FREE OVER-FALL.

(Length of weir, $L = 2$ ft.; entrance rounded on 6-in. radius.)

Test No.	Discharge, in cubic feet per second, Q .	Total head = observed head plus velocity head, H_a .	Coefficient in equation, $Q = CL H_a^{3/2}$, C .	Percentage by which C submerged is greater or less than with free over-fall.
----------	--	--	---	--

PART I: WEIRS WITH SLOPE OF CREST SO SMALL THAT HYDRAULIC JUMP WILL NOT FORM.

SERIES DA.

1	2.603	0.6062	2.757	}	-0.62
2*	2.582	0.6054	2.740		
9	3.291	0.7056	2.776	}	-0.76
8*	3.278	0.7074	2.755		
10	4.135	0.8201	2.784	}	-0.22
11	4.118	0.8189	2.778		
12	5.649	1.0074	2.794	}	-0.21
13*	5.634	1.0070	2.788		
14	10.108	1.4758	2.817	}	-0.53
15*	10.040	1.4752	2.802		
16	8.170	1.2910	2.786	}	+0.32
17*	8.184	1.2892	2.795		
18	6.770	1.326	2.808	}	-0.46
19*	6.775	1.369	2.795		
20	4.922	0.9150	2.812	}	-0.71
21*	4.913	0.9183	2.792		
22	7.577	1.2241	2.798	}	+0.07
23*	7.483	1.2132	2.800		
24	9.189	1.3964	2.785	}	0.00
25*	9.212	1.3986	2.785		
Average for Series DA					-0.31

SERIES DB.

1	2.628	0.6057	2.787	}	-1.04
2*	2.622	0.6092	2.758		
3	4.036	0.8046	2.796	}	-0.11
4*	4.055	0.8077	2.798		
5	5.654	1.0041	2.810	}	-0.18
6*	5.640	1.0035	2.805		
7	7.601	1.2230	2.810	}	-0.43
8*	7.585	1.2249	2.798		
9	9.478	1.4189	2.804	}	+0.18
10*	9.503	1.4196	2.809		
Average for Series DB					-0.32

* Submergence.

TABLE 1.—(Continued.)

Test No.	Discharge, in cubic feet per second, Q .	Total head = observed head plus velocity head, H_a .	Coefficient in equation, $Q = C L H_a^{3/2}$, C .	Percentage by which C submerged is greater or less than with free over-fall.
SERIES DD.				
1	2.670	0.5903	2.943	}
2*	2.671	0.5946	2.913	
3	4.005	0.7755	2.932	}
4*	3.993	0.7753	2.924	
7	5.656	0.9826	2.904	}
8*	5.621	0.9813	2.891	
9	7.429	1.1855	2.878	}
10*	7.369	1.1797	2.876	
11	8.879	1.3377	2.869	}
12*	8.864	1.3343	2.876	
13	2.061	0.4959	2.951	}
14*	2.064	0.4951	2.962	
15	3.423	0.6980	2.935	}
16*	3.415	0.6970	2.934	
17	4.892	0.8884	2.921	}
18*	4.898	0.8900	2.917	
22	2.396	0.5483	2.951	}
24*	2.410	0.5505	2.950	
25	4.396	0.8277	2.919	}
26*	4.427	0.8314	2.920	
27	7.137	1.1529	2.883	}
28*	7.120	1.1527	2.877	
29	8.097	1.2570	2.873	}
30*	8.076	1.2552	2.872	
31	9.318	1.3830	2.865	}
32*	9.321	1.3840	2.862	
Average for Series DD				-0.13
SERIES DG.				
4	2.758	0.6103	2.893	}
5*	2.746	0.6094	2.887	
8	6.781	1.1208	2.857	}
9*	6.810	1.1249	2.855	
12	9.711	1.4308	2.837	}
13*	9.713	1.4336	2.829	
Average for Series DG				-0.19

* Submergence.

TABLE 1.—(Continued.)

Test No.	Discharge, in cubic feet per second, Q .	Total head = observed head plus velocity head, H_a .	Coefficient in equation, $Q = C L H_a^{3/2}$, C .	Percentage by which C submerged is greater or less than with free over-fall.
SERIES D.				
17	5.144	0.9412	2.817	} -0.04
18*	5.258	0.9552	2.816	
Average for thirty-two pairs of tests in Part I				- 0.219
PART II: WEIRS WITH THE HYDRAULIC JUMP FORMED ON THE CREST BY SUBMERGENCE.				
SERIES DE.				
1	4.361	0.8027	3.082	} -0.08
2*	4.361	0.8082	3.081	
3	5.835	0.9780	3.016	} +0.08
4*	5.845	0.9791	3.017	
5	7.385	1.1473	3.005	} -0.33
6*	7.378	1.1492	2.995	
7	8.860	1.8024	2.981	} +0.17
8*	8.856	1.8005	2.986	
10	2.822	0.5988	3.049	} -0.39
11*	2.792	0.5956	3.037	
16	2.513	0.5502	3.079	} -0.19
17*	2.529	0.5533	3.073	
18	4.012	0.7555	3.055	} -0.20
19*	3.998	0.7548	3.049	
20	6.605	1.0600	3.026	} -0.36
21*	6.558	1.0577	3.015	
22	5.229	0.9054	3.085	} +0.16
25*	5.225	0.9039	3.040	
28	5.233	0.9051	3.039	} 0.00
26*	5.224	0.9039	3.039	
24	5.228	0.9035	3.044	} -0.23
27*	5.239	0.9061	3.037	
28	7.546	1.1582	3.027	} -0.36
29*	7.528	1.1592	3.016	
30	8.548	1.2669	2.997	} +0.13
31*	8.547	1.2658	3.001	
32	9.772	1.3880	3.004	} -0.20
33*	9.747	1.3825	2.998	
35	3.122	0.6385	3.060	} +0.26
36*	3.146	0.6406	3.068	
Average for Series DE				-0.10

* Submergence.

TABLE 1.—(Continued.)

Test No.	Discharge, in cubic feet per second, Q .	Total head = observed head plus velocity head, H_a .	Coefficient in equation, $Q = C L H_a^{3/2}$, C .	Percentage by which C submerged is greater or less than with free over-fall.
SERIES <i>DF</i> .				
9	5.025	0.9059	2.914	} -0.38
10*	5.007	0.9060	2.903	
11	6.995	1.1363	2.887	} +0.10
12*	7.011	1.1375	2.890	
13	8.825	1.3274	2.885	} +0.31
14*	8.833	1.3255	2.894	
16	7.642	1.2051	2.888	} +0.10
17*	7.659	1.2060	2.891	
18	2.104	0.5050	2.931	} -0.07
19*	2.092	0.5034	2.929	
20	3.416	0.6987	2.925	} -0.10
21*	3.411	0.6984	2.922	
22	5.947	1.0192	2.890	} -0.21
23*	5.956	1.0217	2.884	
Average for Series <i>DF</i>				-0.04
Average for twenty-two pairs of tests in Part II.....				-0.081
Grand average of fifty-four pairs of tests.....				-0.163

* Submergence.

which the coefficient submerged was more than 0.5% less than the coefficient with free over-fall. If these six pairs of tests are disregarded, the remaining twenty-six pairs give coefficients for the submerged weirs only 0.09% less on the average than for the weirs with free over-fall.

With the jump formed on the crest, the largest difference in coefficients submerged and with free over-fall was 0.39 per cent. Various positions of the jump were tried, but none closer to the entrance of the weir than about 5 ft., at which distance a slight fluctuation in volume of flow merely caused a small movement of the jump up or down stream with no other visible effect on the conditions of flow.

LOCATION OF CRITICAL DEPTH

Profiles of the water surface over the weir for each weir model tested, both submerged and with free over-fall, and with a range of head from 0.5 ft. to 1.5 ft., are shown in the series of graphs in Fig. 4; also the points at which the measured depth equals the computed critical depth for each flow.

The graphs show that none of the weir models tested produced flow at critical depth at any fixed location throughout the entire range of heads. At points where previous discussions had indicated that critical depth might be

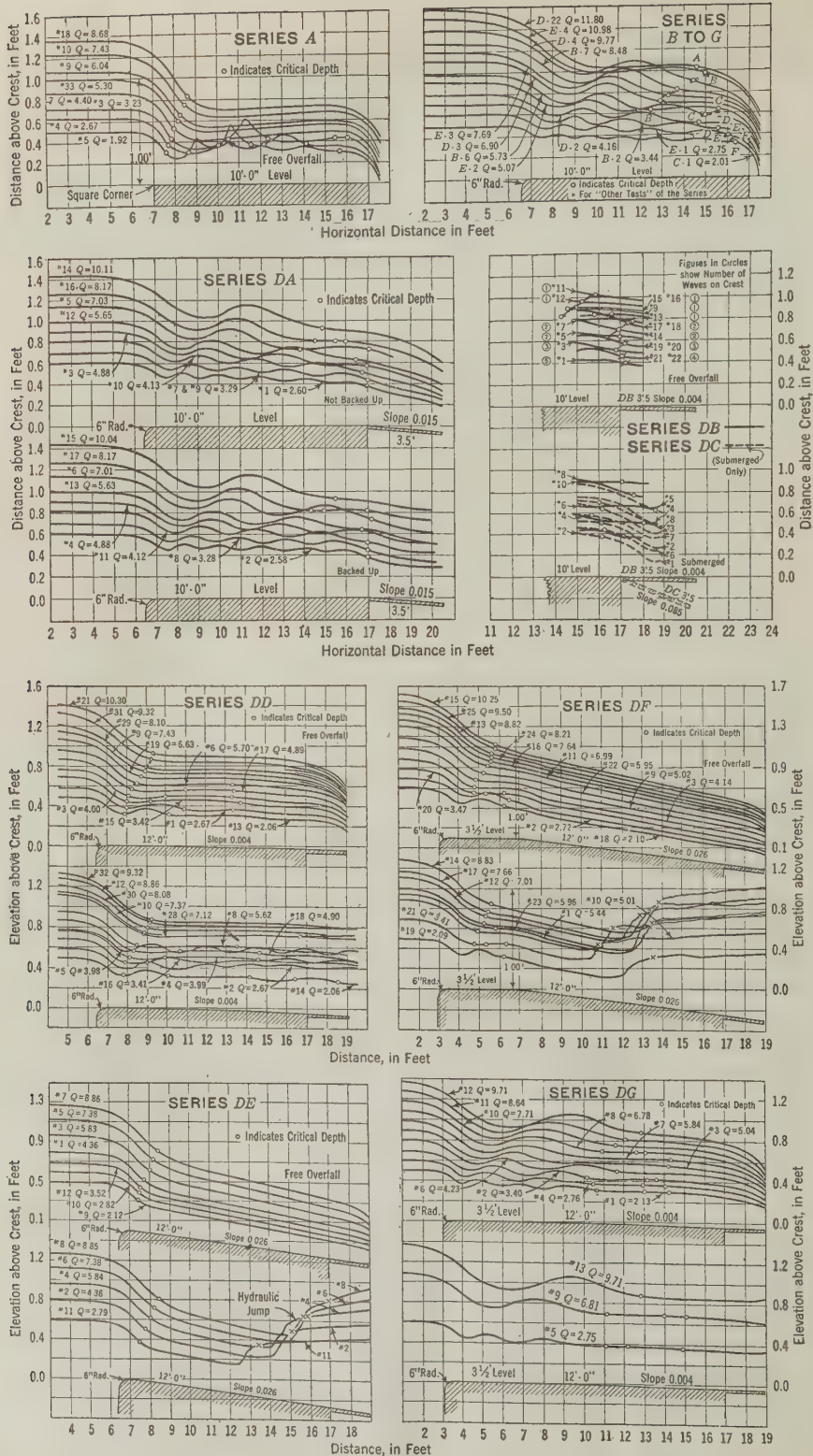


FIG. 4.—PROFILES OF WATER SURFACE AND LOCI OF CRITICAL DEPTH.

expected to occur, as, for instance, at the change from level crest to a small down-stream slope (Series *DA*, *DB*, *DF*, and *DG*), the depth of flow was found to be affected by waves on the crest of the weir. On the other hand, with crest slopes sufficient to maintain flow free of waves (Series *DE*), critical depth occurred, as a rule, only on the initial drop in water surface at the entrance to the weir. One series of tests in which the crest had a down-stream slope of 0.004 (Series *DD*) showed for the smaller flows a depth very near the critical through a distance of 3 or 4 ft. at about the center of the crest; but for flows larger than about 6 cu. ft. per sec., the depth was distinctly less than the critical at all points except on the initial entrance drop.

Although for the models tested no one location was indicated at which critical depth may be expected to occur at any flow, nevertheless, the profiles show some interesting facts in regard to the manner in which it varies with the number of waves on the crest; and also the manner in which the width and slope of the crest, the rounding of entrance, and the volume of flow affect the number and position of the waves.

Waves occurred at certain flows with every weir model tested in which part or all of the crest was level. The profiles of Series *B* to *G* show typical conditions of flow over a weir with a level crest, 10 ft. wide, with rounded entrance. The locus of the points of critical depth in the profiles of this group is a series of disconnected straight lines, apparently with about the same slope, each line containing the points of critical depth accompanying a particular number of waves on the crest. Thus, all profiles showing only one wave on the crest have their points of critical depth on Line *BB*; profiles with two waves have their points on Line *CC*; those with three waves, on Line *DD*; with four waves, on Line *EE*; and with five waves, on Line *FF*. Test No. 6 in Series *B* appears to be a border-line case between one wave and two waves, since its profile shows points of critical depth on both Line *BB* and Line *CC*. Line *AB*, containing critical depth points for flows of about 11 cu. ft. per sec. and greater, appears, from the few tests made at those flows, to slope up to the left instead of up to the right like the other loci.

The same arrangement of the points of critical depth is apparent also in the profiles in Series *DA* and *DB* for free over-all. A majority of the points of critical depth for the lower flows in these two series lie at, or very close to, the change in slope. However, even with these smaller flows, the effect of the waves is apparent in the variations of some of the points from the change in slope. At the larger flows, the points of critical depth all occur from 1 ft. to 3 ft. up stream from the change in slope, and their locus is similar to that of the points on the profiles with one wave in Series *B* to *G*.

The profiles show the variation of the number of waves on the crest with the volume of flow and with the breadth and slope of the crest. At the smallest flows tested (Series *B* to *G*, and *DA*), five distinct waves appear. With increasing flow, the number of waves decreases consecutively until at the largest flows the profiles present a smooth curve free from waves. Moreover, it is apparent that at least within the limits of these tests, the waves on a level crest continue to occur until smoothed out by sufficient increase in the slope

or by a fall at the down-stream end of the level section. Narrowing the weir would thus obviously result in fewer waves at the smaller flows and in a reduction of the volume of flow at which the last wave disappears.

The effect of an increase in slope is shown by the profiles of Series *DF* and *DG*. The weir in Series *DF* consisted of a level section $3\frac{1}{2}$ ft. long, with rounded entrance, followed by 12 ft. of 0.026 slope. Waves occurred singly and only at the two smallest flows. The narrow section of level crest and the steepness of the down-stream slope combined to smooth out the flow quickly. The slope of crest in Series *DG*, however, was not sufficient to smooth out the flow quickly and the profiles show at least one wave for all heads tested and as many as three waves for the smallest flow.

The foregoing discussion of the position of critical depth pertains to the weirs in which the entrance was rounded. With a square-cornered entrance the conformation of the water surface over the weir was quite different, as is seen by comparing the profiles of Series *A* with those of Series *B* to *G*. The initial drop in the water surface was greater with the square-cornered entrance than with the rounded entrance preceding a level crest 10 ft. broad. With the square-cornered entrance the water surface dropped to a point below the critical depth and then commenced to rise gradually. This rise continued either until it approached the critical depth, when the surface became wavy and turbulent, or until it reached the vertical curve caused by the fall. The critical depth was reached and waves were formed only with flows of less than about 4 cu. ft. per sec. The waves formed with the rounded entrance were smooth in profile, and the water surface was practically level in cross-section at all points. With the square-cornered entrance the waves reached more of a peak, being considerably higher in the center of the stream than at the sides; and there was more surface turbulence.

VARIATION OF WEIR COEFFICIENT WITH HEAD

The variation of the weir coefficient, C , with the total head, H_a , in the equation, $Q = CLH_a^{\frac{3}{2}}$, for thirteen different weir models tested, is shown by the series of graphs in Fig. 5. A longitudinal section of the weir model is shown with each graph. The points on the graphs include only tests of weirs with free over-fall.

To facilitate comparisons of the coefficients for the different weir models, all thirteen curves are shown on one graph in Fig. 6; also, two curves obtained by Bazin and three by the U. S. Geological Survey for weirs of the same general type as those reported in this paper.

One important comparison (Fig. 6) is that for the weirs with the broader part of the crest level, with rounded entrance, and either with or without a narrow sloping down-stream apron, the coefficient increases with the head up to values of H_a of about 1.10, after which it is approximately constant (Series *B* to *G*, *DA*, and *DB*); while for weirs with the broader part of the crest sloping down stream, with rounded entrance, and either with or without a narrow level platform preceding the sloping section, the coefficient decreases with the head and shows a tendency to become constant at values of H_a of about 1.30 (Series *DD*, *DE*, *DF*, and *DG*).

Series *B* to *G* (Fig. 6) also indicate that for the breadth of weirs tested, varying the curvature of the weir entrance between a 2-in. and an 8-in. radius has practically no effect on the weir coefficient. The graphs obtained with six different curvatures—four circular and two elliptical—all lie within 0.5% of one another at all points. The graph for a square-cornered entrance (Series *A*), however, is considerably lower, showing values of the coefficients from 6 to 8% less than for rounded entrances. Apparently, the entire advantage of rounding the entrance is obtained with the change from the square corner to the 2-in. radius, and further increase in the radius does not appreciably increase the coefficient.

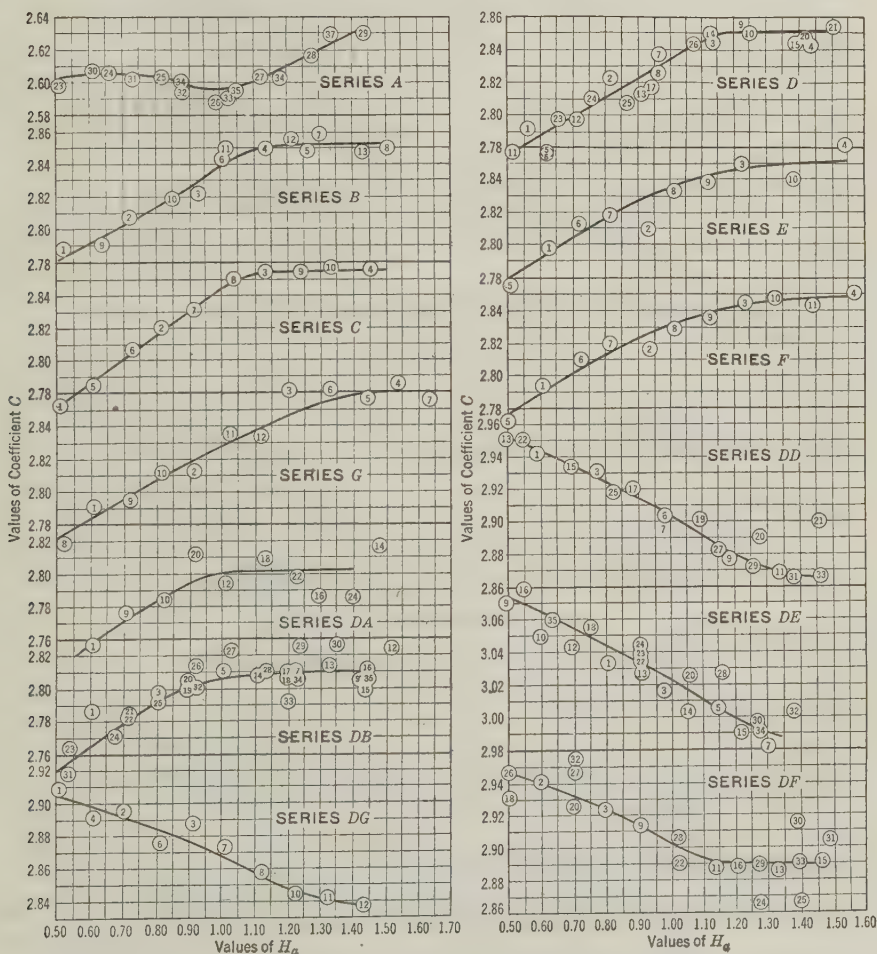
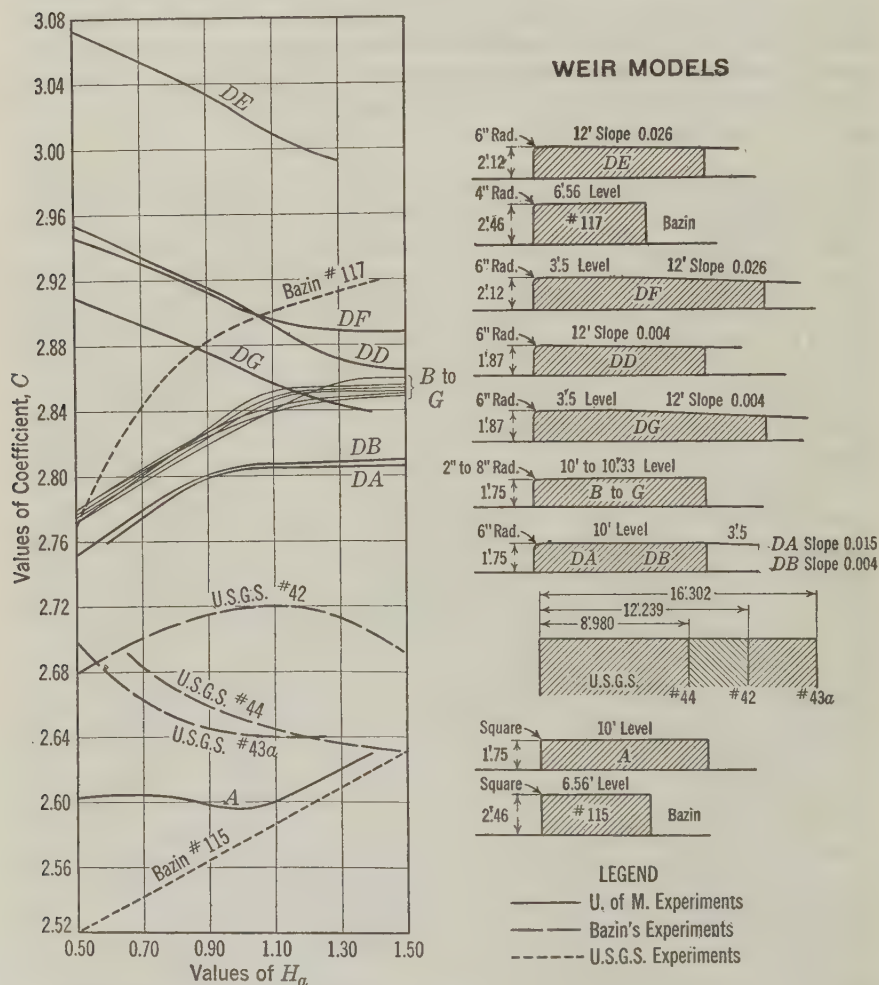


FIG. 5.—VARIATION OF COEFFICIENT OF DISCHARGE, C , WITH TOTAL HEAD, H_a , IN FORMULA, $Q = C L H_a^{3/2}$. H_a = OBSERVED HEAD + VELOCITY HEAD; $L = 2.0$ FEET.

A definite increase in coefficient with increase in slope of the crest is also shown. A comparison of the curves of Series *DE* and *DD* for the same width

of crest, but with different slopes, shows that the coefficients for the weir with a crest slope of 0.026 are about 4% higher throughout the range of heads tested than the coefficients for the weir with a crest slope of only 0.004. Tests in Series *DF* and *DG*, in which a 3½-ft. level section preceded the sloping



OBSERVED HEAD - VELOCITY HEAD

FIG. 6.—VARIATION OF COEFFICIENT OF DISCHARGE, C , WITH TOTAL HEAD, H_a . BAZIN CURVE AND U. S. GEOLOGICAL SURVEY CURVES ARE TAKEN FROM WATER SUPPLY PAPER No. 200, PLATES IV, XXIX, AND XXX.

section, also show higher coefficients for the weir with steeper slope. The difference is not so great, however, as in the narrower weirs of Series *DE* and *DD*, indicating that a greater width tends to reduce the effect of a change in slope.

The tests show a reduction in coefficient with an increase in the width of the weir. Comparison of Series *DA* and *DB* with Series *B* to *G* shows that a decrease of about 1% in the coefficient resulted when a 3½-ft. downstream apron with a slight slope was added to the weir with level crest. Moreover, comparison of Curves *DE*, *DF*, and *DD* shows that the coefficients were reduced about as much by the addition of 3½ ft. of level section ahead of the sloping section as by reducing the slope from 0.026 to 0.004. The height of these weirs varied from 1.75 to 2.12 ft., but there is little probability that the conclusions are appreciably in error on this account, especially since the variation in height was largely compensated for by including the correction for velocity of approach in the head used to compute the weir coefficients.

All the tests show fairly good agreement with those obtained by other experimenters on weirs of the same general type. The results of Series *B* to *G* are comparable with Bazin's Test No. 117 (Fig. 6) on a weir with level crest, 6.56 ft. broad, with rounded entrance. As might be expected with a narrower weir, the curve of Bazin's results lies somewhat higher than the curves of Series *B* to *G*. For a sharp-cornered entrance, the curve of Series *A* for a weir with a crest 10 ft. broad agrees in general location with the curves of Bazin's Test No. 115 and of the U. S. Geological Survey Tests Nos. 42, 43A, and 44 (Fig. 6).

An inspection of the individual graphs of Fig. 5 shows that in 201 tests of the 13 weirs, only one point, No. 21 in Series *DD*, is more than 1% off from its curve, while the large majority are well within 0.5% of the curve.

Since H_a enters the broad-crested weir equation to the three-halves power, an error of 0.5% in the computed value of C is caused by an inverse error of approximately one-third of 1% in observing the head on the broad-crested weir. At the lowest head tested, 0.5 ft., this error would be produced by an actual error in reading the head of 0.0017 ft.; while at the highest head, 1.5 ft., the error in reading the head would have to be only 0.005 ft. to produce an error of 0.5% in the coefficient.

CONCLUSIONS

The tests furnished experimental verification of the theory that a broad-crested weir can be properly designed to operate submerged with small loss of head, and with the same coefficients as for a weir of the same design with free over-fall. Measurement of head, moreover, is required at only one point. The minimum loss of head occurs when the weir is designed to produce a hydraulic jump on the crest. As long as the jump does not approach too close to the weir entrance, its position does not affect the coefficient. Proper design of the weir, therefore, can provide for considerable fluctuation in discharge, with resulting variation in position of the jump, without change in the head-water-discharge relation.

In addition, the tests brought out difficulties which must be overcome in producing flow at critical depth at some predetermined point on the crest of the weir. The chief difficulty is the formation of waves on the crest which affect the location of critical depth. Tests of other types of entrance with different alignment of the sides of the channel may lead to a design of weir

which will maintain the flow at the desired depths without these disturbing waves. The measurement of flowing water with a broad-crested weir of this type would require only a determination of the depth at the predetermined point and would not involve velocity of approach, which has been a continued source of uncertainty in measurements with sharp-crested weirs.

PERSONNEL

The tests were performed by the writer under the leadership and guidance of H. W. King, M. Am. Soc. C. E., and with the assistance of C. O. Wisler, Assoc. M. Am. Soc. C. E., and Messrs. H. A. Seaman, J. G. Jobes, C. L. Palmer, A. J. Paddock, P. Singh, and C. Li, graduate students and seniors at the University of Michigan. Most of the drawings were by Mr. Seaman.

APPENDIX

SUPPLEMENTAL TESTS—WEIRS WITH APRONS INCLINED UP STREAM AND DOWN STREAM

BY ALEX. R. WEBB,* M. AM. SOC. C. E.

SCOPE

This Appendix describes experiments conducted in continuation of Mr. Woodburn's work on broad-crested weirs. The object was to determine if possible a form of weir crest which would cause the locus of critical depths to lie in a vertical line. The general dimensions of the weir used are illustrated in Fig. 7. The apex of the two sloping aprons forms the highest section of the weir crest.

Results are given for 235 separate experiments. The down-stream apron was set at slopes of 1 in 15.4, 1 in 24.8, 1 in 25.0, 1 in 25.5, and 1 in 46.4. The slope of the up-stream apron was varied between 1 in 7.5 and 1 in 98.5. The heads ranged from 0.12 to 1.33 ft. and the discharges from 0.22 to 9.33 cu. ft. per sec.

The experiments show that for the flatter slopes waves formed on the up-stream apron and that the location of critical depth changed with the position of the wave. This condition was less marked for the smaller discharges. Except for these very flat slopes it was found that for each slope of the down-stream apron there was a corresponding slope of the up-stream apron, which caused critical depths for all discharges to lie in a vertical straight line near, but not necessarily above, the apex of the crest.

The effect of submergence on the relation of head to discharge was investigated by making parallel runs both with and without submergence. In all, fourteen pairs of comparative tests were made. The value of the coefficient, C , in the weir formula, $Q = C L H_a^{\frac{3}{2}}$, was also investigated. In addition, a series of experiments was conducted to check the formula for the V-notch weir which was used to measure discharges.

* Prof., Civ. Eng., Ohio Northern Univ., Ada, Ohio.

LABORATORY

With slight modification the equipment used is the same as that described in the paper. The horizontal broad-crested weir was modified by building a small bulkhead at the up-stream end and attaching the sloping aprons, shown in Fig. 7. The down-stream apron, the false bottom referred to by Mr. Woodburn, was fixed for each set of experiments. The up-stream apron was attached by hinges to the bulkhead so that the slope could be easily changed.

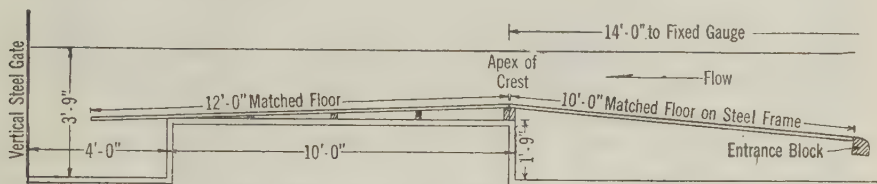


FIG. 7.—LONGITUDINAL SECTION OF WEIR.

During the first experiments, Series WA to WH, uplift pressure caused the floor to pull away from the joists, forming a slightly convex surface instead of a plane. Thereafter, matched flooring attached to a welded steel frame ensured a plane surface under all conditions.

For all experiments except Series WA, WB, and WC, a 3 by 6-in. elliptical laminated block was bolted to the up-stream apron to give a rounded entrance.

TEST PROCEDURE

An outline of conditions to which the experiments conform is given in Table 2. In making each run, the profile of water surface over the weir was determined from observations 2 ft. or less apart. At the time of taking readings along the center line of the channel, simultaneous readings were made for the fixed gauge and the V-notch-weir gauge. Six readings were taken across the apex of the weir while simultaneous readings were being taken at the fixed gauge. While these observations were being made, as many readings as possible of the V-notch-weir gauge were taken. (See Table 3.) One reader attended each of the three gauges, and one man acted as recorder for the fixed gauge and the movable gauge.

ANALYSIS OF RESULTS

The results of the investigation will be discussed under several heads.

Location of Critical Depth.—A profile of water surface was plotted for each run. Usually 6 to 8 runs (constituting a series of experiments) were conducted for each setting of the aprons. In all there were 37 series. The point of critical depth was determined from the formula, $Q = \sqrt{g} L D_c^{\frac{3}{2}}$, or, for length of weir, $L = 2$, $D_c = 0.1981 Q^{\frac{2}{3}}$. This depth was plotted on each profile.

Profiles for each series of experiments were plotted on a single sheet. Typical examples are shown in Fig. 8. Five sets of experiments containing, respectively, 8, 10, 9, 6, and 4 series were performed. In each set the slope of the down-stream apron was kept constant and that of the up-stream apron was varied. Profiles of each set of experiments plotted to a larger scale on a

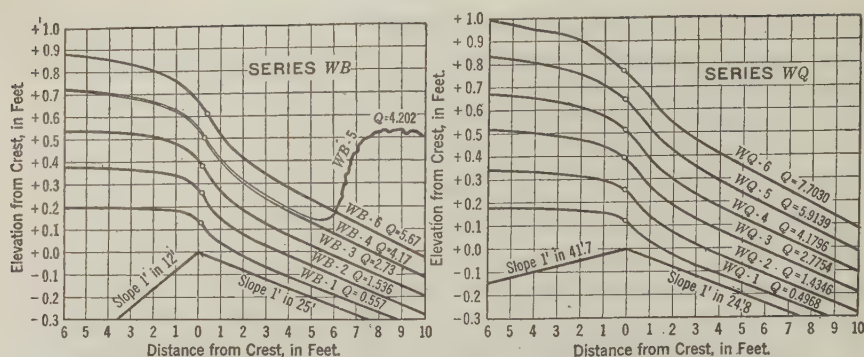


FIG. 8.—EXAMPLES OF WATER PROFILES, SHOWING CRITICAL DEPTHS.

single diagram and showing water surfaces 1.5 ft. up stream and down stream from the apex (Fig. 9), indicate that while there are individual variations, in general the points of critical depth lie on well-defined curves. These curves plotted to a greatly distorted scale are shown in Fig. 10.

TABLE 2.—OUTLINE OF EXPERIMENTS.

(Series WA, WB, and WC were made without the use of the block for rounded entrance. The wooden frame of the up-stream apron was changed to a welded steel frame after Series WH.)

Series.	Number of runs.	Up-stream slope.	Down-stream slope.
WA	6	1 in 7.4	1 in 25
WB	6	1 in 12	1 in 25
WC	3	1 in 46.8	1 in 25
WD	6	1 in 46.8	1 in 25
WE	13	1 in 31.5	1 in 25
WF	12	1 in 26.2	1 in 25
WG	8	1 in 22.2	1 in 25
WH	15	1 in 30 to 32	1 in 25
WI	15	1 in 25.6	1 in 24.8
WJ	16	1 in 24.0	1 in 24.8
WK	7	1 in 24.45	1 in 24.8
WL	9	1 in 27.1	1 in 24.8
WM	5	1 in 30.3	1 in 24.8
WN	6	1 in 33	1 in 24.8
WO	5	1 in 34.8	1 in 24.8
WP	5	1 in 39.8	1 in 24.8
WQ	6	1 in 41.7	1 in 24.8
WR	6	1 in 50.2	1 in 24.8
WA A	8	1 in 40.3	1 in 25.5
WA B	5	1 in 37.6	1 in 25.5
WA C	6	1 in 42.3	1 in 25.5
WA D	6	1 in 47.1	1 in 25.5
WA E	6	1 in 34.1	1 in 25.5
WA F	6	1 in 29.8	1 in 25.5
WA G	6	1 in 40.0	1 in 25.5
WA H	6	1 in 41.3	1 in 25.5
WA I	6	1 in 40.5	1 in 25.5
BA	5	1 in 40.9	1 in 25.5
BB	6	1 in 50.0	1 in 46.4
BC	5	1 in 65.1	1 in 46.4
BD	4	1 in 80.5	1 in 46.4
BE	6	1 in 89.55	1 in 46.4
BF	6	1 in 98.5	1 in 46.4
CA	4	1 in 40.2	1 in 15.37
CB	8	1 in 30.15	1 in 15.37
CC	5	1 in 19.95	1 in 15.37
CD	4	1 in 24.99	1 in 15.37

TABLE 3.—SERIES WP-5: GAUGE READINGS AND HEADS FOR DETERMINING DISCHARGE, CRITICAL DEPTH, AND PROFILE OF WATER SURFACE.

V-NOTCH WEIR.		FIXED GAUGE.		Point on weir.	MOVABLE GAUGE.				Eleva- tion above crest, in feet.
Observed readings, in feet.	Head, in feet.	Observed readings, in feet.	Head, in feet.		Observed Readings, in Feet.				
					Left.	Center.	Right.	Average.	
1.157	1.102	3.796	0.650	10		0.762			-0.172
1.157	1.102	3.797	0.651	8		0.856			-0.078
1.157	1.102	3.796	0.650	6		0.956			+0.022
1.156	1.101	3.796	0.650	4		1.041			0.107
1.158	1.103	3.796	0.650	3		1.096			0.162
1.158	1.103	3.797	0.651	2	1.152	1.154	1.158	1.155	0.221
1.159	1.104	3.798	0.652	1.5	1.189	1.189	1.198	1.192	0.258
1.158	1.103	3.798	0.652	1.0	1.239	1.235	1.242	1.239	0.305
1.158	1.103	3.797	0.651	0.5	1.294	1.293	1.296	1.294	0.360
1.1581*	1.1036	3.7972	0.6517	0.0	1.353	1.354	1.352	1.3527	0.4188
				0.0	1.353	1.352	1.352		
1.158	1.103	3.796	0.650	0.5	1.393	1.395	1.394	1.394	0.460
1.158	1.103	3.797	0.651	1.0	1.416	1.417	1.421	1.418	0.484
1.156	1.101	3.795	0.649	1.5	1.435	1.435	1.440	1.437	0.508
1.156	1.101	3.795	0.649	2	1.448	1.447	1.450	1.448	0.514
1.155	1.100	3.796	0.650	3		1.468			0.534
1.154	1.099	3.796	0.650	4		1.482			0.548
1.155	1.100	3.795	0.649	5		1.494			0.560
1.156	1.101	3.797	0.651	6		1.505			0.571
Average..	1.1025		0.6509						

* Average of 12 readings.

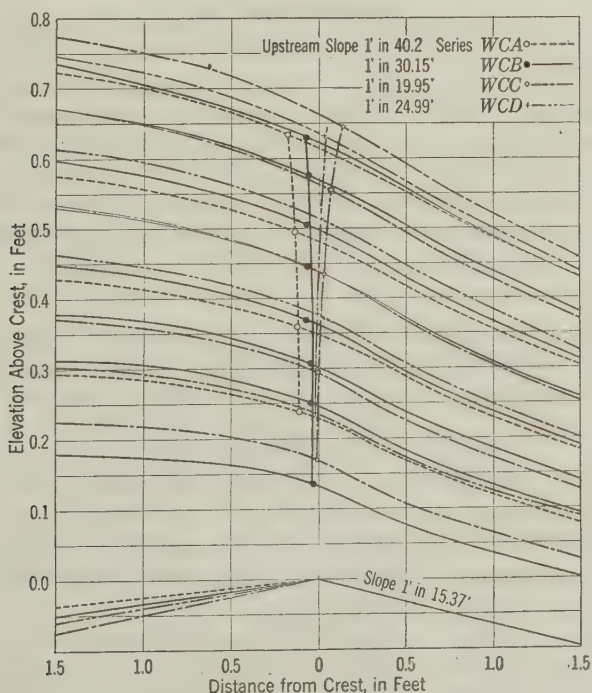


FIG. 9.—ENLARGED PROFILES, SHOWING CRITICAL DEPTHS.

In one series of experiments, where the slope of the down-stream apron was 1 in 25.5 and that of the up-stream apron, 1 in 40.3, the points of critical depth for all discharges fell on a vertical straight line over the apex. This condition was not duplicated for any other combination of slopes. With down-stream and up-stream slopes, respectively, 1 in 24.8 and 1 in 39.8, critical depths fell in a vertical line, 0.08 ft. up stream from the apex. Experiments on a much steeper down-stream slope (1 in 15.4) indicate that critical depths would lie in a vertical line about 0.04 ft. up stream from the apex when the up-stream slope was about 1 in 28.

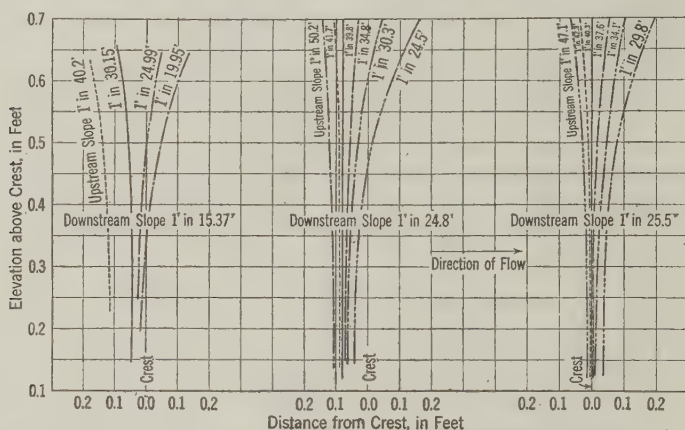


FIG. 10.—VARIATIONS IN LOCI OF CRITICAL DEPTHS CORRESPONDING TO SLOPES OF WEIRS.

In all cases where the up-stream apron was given a very flat slope, waves formed on the crest of the weir. The position of critical depth then became variable, depending more or less on the location of the waves. The condition for an up-stream slope of 1 in 98.5 is illustrated in Fig. 11(a). The tendency of waves to form on the up-stream apron was less marked for the smaller discharges. There were slight indications of waves for slopes of about 1 in 50. No waves were observed when the slope of the up-stream apron was 1 in 40, or less.

Calculations were made to determine the percentage of error that would result from computing discharges by the formula, $Q = \sqrt{g} L D_c^{3/2}$, substituting the measured depth over the apex D_a . The results of the computations for Series W4.4 and WP, in which the critical depths were approximately in verticals, over the apex and 0.08 ft. up stream from the apex, respectively, are shown in Table 4. In the first case the percentages of error that would result from using the measured depth over the apex as the critical depth, lie between the limits, -0.7% and $+0.5$ per cent. In the second case the corresponding percentages range from -2.1 to -7.4 . These results show clearly the sensitive character of this type of structure.

Effect of Submergence on Head-Discharge Relation.—One of the advantages of weirs with broad crests is that the down-stream water surface may be at an elevation materially higher than the crest of the weir without affecting the relation between head and discharge. Weirs having this characteristic per-

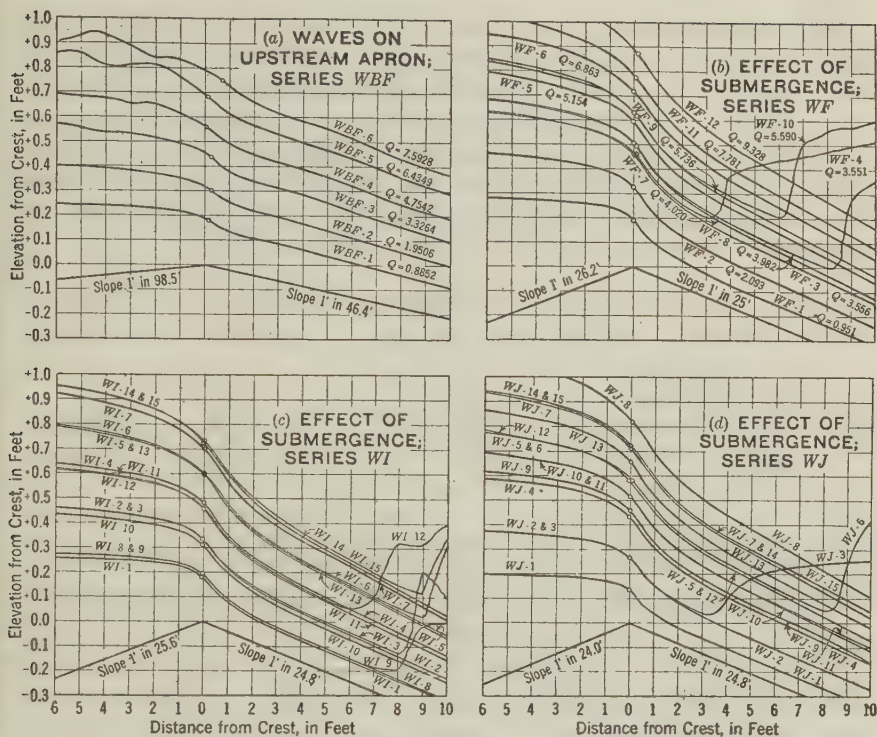


FIG. 11.—EXAMPLES OF WATER PROFILES AND INFLUENCE OF WAVES.

mit the measurement of discharges without great loss of head. Fourteen pairs of experiments were conducted to study the effects of submergence. In so far as practicable the discharge for both runs of any pair was kept constant. One run was made with free discharge and the other with the water backed up so as to cause a hydraulic jump on or just below the down-stream apron. Since the down-stream end of the apron was only 2 ft. from the outlet of the flume, it was not possible to determine the effect of a jump farther down stream.

TABLE 4.—EFFECT UPON CALCULATED DISCHARGE OF USING DEPTH OVER APEX INSTEAD OF CRITICAL DEPTH.

Series No.	Discharge, $Q = 2.505 H^{2.48}$	Critical depth, $D_c = 0.1981 Q^{3/8}$	Measured depth at crest = D	$D - D_c$	Calculated discharge, $Q' = \sqrt{g L D^{3/2}}$	Error, $Q' - Q$	Percent- age.
W A A-3	0.4984	0.1246	0.1250	+0.0004	0.5006	+0.0022	+0.44
W A A-4	1.4377	0.2523	0.2515	-0.0008	1.4300	-0.0077	-0.53
W A A-5	2.7604	0.3898	0.3880	-0.0018	2.7410	-0.0194	-0.70
W A A-1	3.1153	0.4226	0.4231	+0.0005	3.1211	+0.0065	+0.21
W A A-4	4.0544	0.5038	0.5055	+0.0017	4.0750	+0.0206	+0.51
W A A-7	5.8270	0.6416	0.6405	-0.0010	5.8129	-0.0141	-0.24
W A A-2	6.1185	0.6628	0.6626	-0.0002	6.1157	-0.0028	-0.05
W A A-8	8.0123	0.7933	0.7933	0.0000	8.0123	0.0000	0.00
W P-5	0.6040	0.1416	0.1346	-0.0070	0.5594	-0.0446	-7.40
W P-4	1.8595	0.2295	0.2906	-0.0089	1.7764	-0.0831	-4.47
W P-3	3.1982	0.4300	0.4183	-0.0117	3.0678	-0.1304	-4.09
W P-2	5.1040	0.5573	0.5784	-0.0089	4.9875	-0.1165	-2.28
W P-1	7.5400	0.7619	0.7509	-0.0110	7.3788	-0.1612	-2.14

In all these experiments it was found that backing up the tail-water caused a slightly increased depth over the apex and a slight increase in head at the fixed gauge. Table 5 gives comparative discharges for each pair of experiments computed from the formula, $Q = \sqrt{g} L D^{\frac{3}{2}}$, D being the depth over the apex. These values compared with measured discharges show that submergence affects the discharge by 0.31 to 2.81 per cent. In all the experiments the slope of the down-stream apron was approximately 1 in 25, but the slope of the up-stream apron varied from 1 in 7.5 to 1 in 30. (See Fig. 11.)

TABLE 5.—SHOWING EFFECT OF SUBMERGENCE UPON DEPTH AND DISCHARGE.

Series No.	Discharge, $Q = 2.505 H^{\frac{3}{2}}$.	Critical depth, $D_c = 0.1981 Q^{\frac{2}{3}}$.	Measured depth, at crest = D .	$D - D_c$.	Calculated discharge, $Q' = \sqrt{g} L D^{\frac{3}{2}}$.	Error, $Q' - Q$.	Percentage.	Increase, in percentage.	Extent of submergence.
W I- 8	0.9657	0.1934	0.1896	-0.0038	0.9372	-0.0285	-2.96	-0.12	Beyond apron.
W I- 9	0.9633	0.1931	0.1891	-0.0040	0.9336	-0.0297	-3.08		
W J- 2	1.5558	0.2660	0.2621	-0.0039	1.5217	-0.0341	-2.19	0.47	4 ft. beyond crest.
W J- 3	1.5465	0.2649	0.2619	-0.0030	1.5200	-0.0265	-1.72		
W I- 2	2.2078	0.3359	0.3319	-0.0040	2.1682	-0.0396	-1.80	0.45	7 ft. beyond crest.
W I- 3	2.2078	0.3359	0.3329	-0.0030	2.1780	-0.0298	-1.35		
W I-11	3.5088	0.4574	0.4534	-0.0040	3.4633	-0.0455	-1.29	1.87	8 ft. beyond crest.
W I-12	3.4566	0.4528	0.4546	+0.0018	3.4767	+0.0201	+0.58		
W F- 3	3.5556	0.4615	0.4597	-0.0018	3.5335	-0.0221	-0.62	1.86	3 ft. beyond crest.
W F- 4	3.5508	0.4611	0.4649	+0.0038	3.5944	+0.0436	+1.24		
W F- 7	4.0195	0.5009	0.5014	+0.0005	4.0244	+0.0049	+0.12	1.85	8 ft. beyond crest.
W F- 8	3.9820	0.4978	0.5035	+0.0057	4.0511	+0.0691	+1.73		
W A- 4	3.9266	0.4981	0.5480	+0.0549	4.6000	+0.6734	+17.14	1.58	4 ft. beyond crest.
W A- 5	4.0102	0.5001	0.5607	+0.0606	4.7613	+0.7513	+18.72		
W B- 4	4.1699	0.5134	0.5422	+0.0288	4.5263	+0.3564	+8.55	1.28	7 ft. beyond crest.
W B- 5	4.2021	0.5160	0.5492	+0.0332	4.6150	+0.4129	+9.83		
W J-10	4.2434	0.5194	0.5121	-0.0073	4.1550	-0.0884	-2.08	0.99	Beyond apron.
W J-11	4.2224	0.5177	0.5139	-0.0048	4.1763	-0.0461	-1.09		
W J- 5	5.0850	0.5821	0.5771	-0.0050	4.9713	-0.0637	-1.27	1.54	9 ft. beyond crest.
W J- 6	5.0340	0.5820	0.5831	+0.0011	5.0475	+0.0135	+0.27		
W I- 5	5.2280	0.5967	0.5961	-0.0006	5.2200	-0.0080	-0.15	0.62	10 ft. beyond crest.
W I- 6	5.2690	0.5999	0.6018	+0.0019	5.2988	+0.0248	+0.47		
W F- 9	5.7360	0.6349	0.6330	-0.0019	5.7113	-0.0247	-0.43	2.81	6 ft. beyond crest.
W F-10	5.5900	0.6241	0.6339	+0.0091	5.7229	+0.1329	+2.38		
W J-14	6.9138	0.7190	0.7156	-0.0034	6.8643	-0.0495	-0.72	0.31	Beyond apron.
W J-15	6.9842	0.7238	0.7219	-0.0019	6.9557	-0.0285	-0.41		
W I-14	7.0235	0.7266	0.7204	-0.0062	6.9343	-0.0892	-1.27	1.48	Beyond apron.
W I-15	7.0168	0.7261	0.7271	+0.0010	7.0314	+0.0146	+0.21		

THE VARIATION OF WEIR COEFFICIENT WITH HEAD AND SLOPE

Values of the coefficient, C , in the weir formula, $Q = C L H_a^{\frac{3}{2}}$, show a wide variation for comparatively small changes in discharges and in slopes of aprons. This variation is indicated quite clearly in Fig. 12. In determining Q and H , gauge readings for a complete run were averaged.

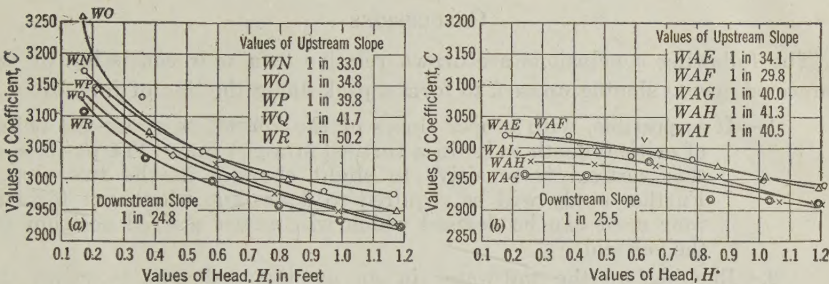


FIG. 12.—VARIATION IN VALUES OF COEFFICIENT, C .

CHECKING V-NOTCH WEIR FORMULA

Preliminary to beginning the experiments, an examination of the V-notch weir used by Mr. Woodburn for determining discharges, showed the face of the weir to be quite heavily coated with mineral deposits. A series of experiments was made with the weir in this coated condition and another series after it had been cleaned. The quantity of flow was obtained by weighing as described in the paper. The results of these experiments (Table 6) show average discrepancies with Mr. Woodburn's formula before and after cleaning, of 0.44 and 0.33%, respectively. In view of this close agreement Mr. Woodburn's formula was accepted as being substantially correct.

TABLE 6.—EFFECT OF DEPOSITS ON FACE OF V-NOTCH WEIR.

Head on V-notch weir, in feet.	Discharge obtained by weighing.	Discharge computed by formula, $Q = 2.505 H^{2.48}$.	Percentage in error.
BEFORE CLEANING.			
0.83156	1.5664	1.5862	+1.26
0.93995	2.1256	2.1488	+1.10
1.05436	2.8309	2.8570	+0.92
1.15504	3.5636	3.5808	+0.48
1.15995	3.6118	3.6197	+0.22
1.16639	3.6546	3.6693	+0.40
1.26225	4.4590	4.4634	+0.10
1.39642	5.7543	5.7342	-0.35
1.39914	5.7743	5.7617	-0.21
Average.....			+0.44
AFTER CLEANING.			
1.15604	3.5688	3.5893	+0.54
1.16309	3.6197	3.6426	+0.64
1.23025	4.1533	4.1892	+0.87
1.22749	4.1545	4.1657	+0.15
1.29310	4.7165	4.7390	+0.48
1.29335	4.7234	4.7411	+0.38
1.39090	5.6864	5.6779	-0.15
1.39220	5.7041	5.6910	-0.24
Average.....			+0.33

CONCLUSIONS

The following conclusions are drawn relative to a weir consisting of two connected aprons sloping upward to form a peak along the line of junction:

- 1.—It is possible, with proper slopes of the aprons, to cause the points of critical depth to lie in a vertical straight line. The position of this vertical is sensitive to slight changes in the two slopes. Further study will be required to determine whether a form of weir crest can be devised which will assure a fixed position for the vertical.
- 2.—Backing up the tail-water in an amount sufficient to cause the hydraulic jump to occur upon the apron increases the depth of water on the weir.
- 3.—Values of the coefficient, C , in the weir formula, $Q = CLH_a^{\frac{3}{2}}$, are very sensitive to small changes in the slopes of the aprons.

ACKNOWLEDGMENTS

The experiments were performed under the direction of H. W. King, M. Am. Soc. C. E. Harry Larson, Assoc. M. Am. Soc. C. E., collaborated in conducting the experiments and in computations. The following students in the College of Engineering, University of Michigan, assisted in the laboratory, Messrs. E. V. Schulhauser, V. K. Ipe, M. Mortenson, D. W. Richardson, I. Curtis, F. F. Smalla, and C. O. Bloom.

During the past year (1929-30) the Society's Special Committee on Irrigation Hydraulics has co-operated in the experiments. In particular, D. C. Henny, and J. C. Stevens, Members, Am. Soc. C. E., Chairman and Secretary, respectively, of the Committee have made valuable suggestions.

